# Engineering GatorTRAX

## Projectile Motion Module
## Intermediate Level

*Designed in accordance with Tau Beta Pi MindSET standards*
*By* ***University of Florida Engineering Ambassadors, 2009***

**Angles**

<u>Definition</u>: A measure of how far one line is rotated off of another line. Angles are measured in degrees or radians. Radians have no units. The conversion between degrees and radians involves a number called pi that is about equal to 3.14. The formula for the conversion is below. We will work in degrees during most of this lesson

Radians = Degrees*pi/180

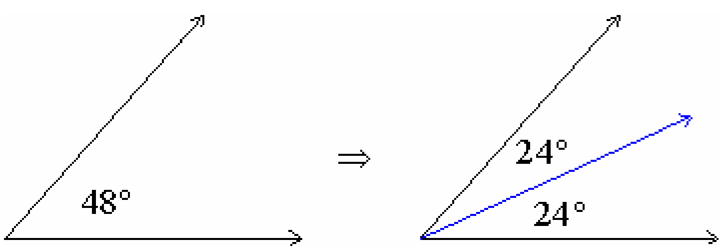
Angles are important to us today for positioning purposes. Our rockets will travel further if they are launched from the appropriate angle. It has been proven that the best angle for today's practices is 45°.

## I. Angle Bisector

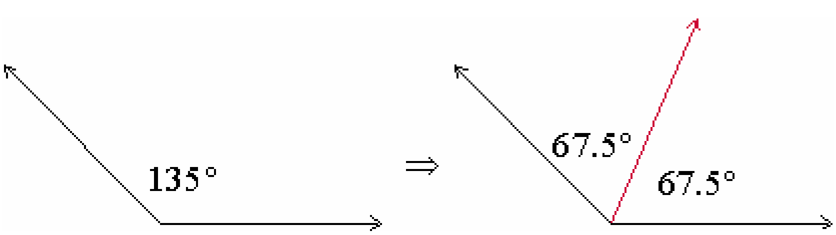
An angle bisector is a ray that divides an angle into two equal angles.

Example:

The blue ray on the right is the angle bisector of the angle on the left.

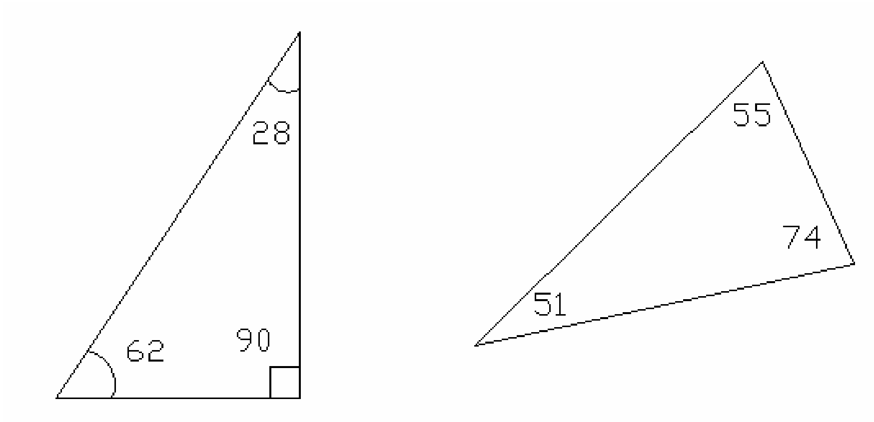



The red ray on the right is the angle bisector of the angle on the left.



**Triangles**

Definition: A closed figure with three sides. All triangles have 180° in them when you add up the three angles. The largest angle is always opposite the longest side and the smallest angle is always opposite the smallest side.




**Types of Triangles**

Right Triangle: a triangle that has one 90°angle in it. A triangle can have no more than one 90° angle.

Acute Triangle: a triangle with every angle less than 90°.

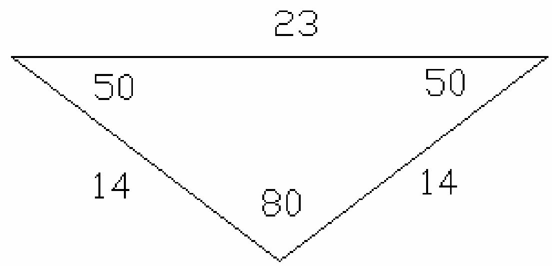
Obtuse Triangle: a triangle that has one angle greater than 90°. The other two angles would then have to be less than 90° when added together.

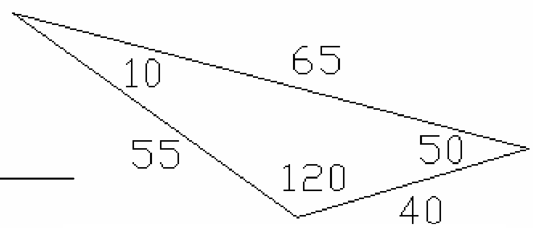
Scalene Triangle: a triangle that has all three sides not equal to each other in length. They also have three different angles

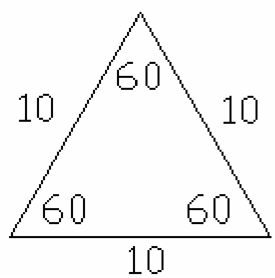
Equilateral Triangle: a triangle with every side equal to the same length which means every angle is also the same. Each angle will be 60°.

Isosceles Triangle: a triangle with two sides of the same length which means they also have two angles the same (the ones opposite of the two equal sides).

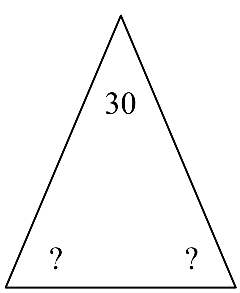
**Can you identify what type of triangles are featured below?**




**Question:** An isosceles triangle has angle A 30 degrees greater than angle B. Find all angles of the triangle.

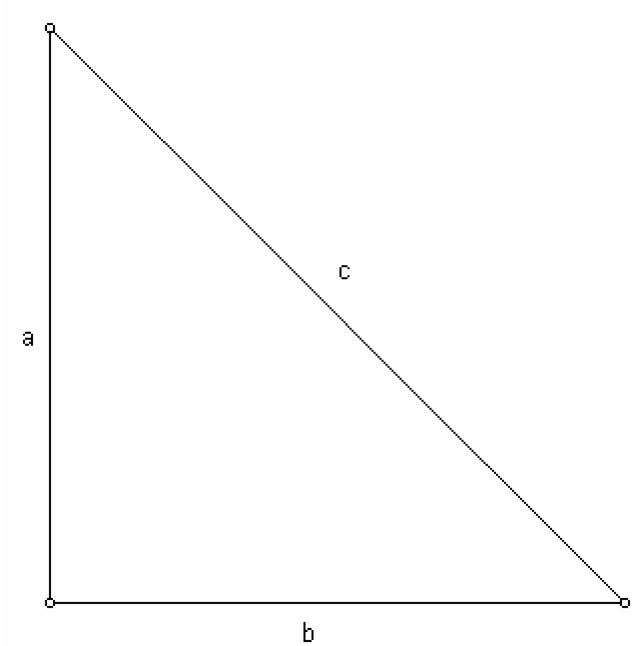


## Pythagorean Theorem

The Pythagorean Theorem states:

**$a^2 + b^2 = c^2$**

for a right triangle with legs *a* and *b* and hypotenuse *c*.




**Example Problems:**

Use this figure to find the missing side length. (There is a right angle between sides *a* and *b*.)

**1. If *a* = 3 and *b* = 4, find the length of *c*.**

$a^2 + b^2 = c^2$
$3^2 + 4^2 = c^2$
$9 + 16 = c^2$
$25 = c^2$
$5 = c$

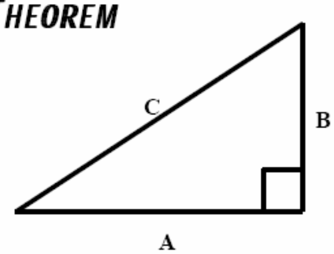
**2. If *a* = 4 and *c* = 11, find the length of *b*.**

$a^2 + b^2 = c^2$
$4^2 + b^2 = 11^2$
$16 + b^2 = 121$
$b^2 = 105$
$b = 10.25$

**Solve:**

### RIGHT TRIANGLES & PYTHAGOREAN THEOREM

1) Solve for the missing side:




a) A = 2
B = 2
C = _____

b) A = 4
B = 2
C = ______

c) A = 4
B = _____
C = 5

d) A = 5
B = ______
C = 12

e) A = _____
B = 6
C = 25

f) A = ______
B = 15
C = 20

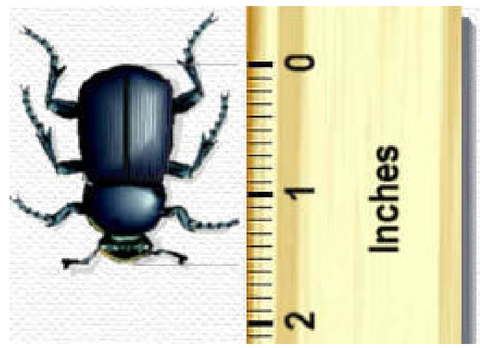
**Measurements**

This lesson covers some of the larger U.S. length units including:

- foot
- yard=3 feet
- eighth mile = 220 yards = 660 feet
- quarter mile = 440 yards = 1320 feet
- half mile = 880 yards = 2640 feet
- mile = 1760 yards = 5280 feet

There are other length units such as rods, chains, leagues, nautical miles, hands, etc. but they are used only for specialized areas or measurements.

**Which of the following best describes the length of the beetle's body in the picture to the right?**




- ☐ Between 0 and 2 in
- ☐ Between 1 and 2 in
- ☐ Between 1.5 and 1.6 in
- ☐ Between 1.54 and 1.56 in
- ☐ Between 1.546 and 1.547 in

The metric system has prefix modifiers that are multiples of 10.

- A kilometer (km) is 1000 meters
- A hectometer (hm) is 100 meters
- A decameter (dam) is 10 meters
- A meter (m) is the basic unit of length
- A decimeter (dm) is 1/10 meter
- A centimeter (cm) is 1/100 meter
- A millimeter (mm) is 1/1000 meter

As we move down the units, the next unit is one tenth as long. As we move upward, each unit is 10 times as long. One hundred millimeters, which is 1/10 meter (100/1000=1/10) are larger than one centimeter (1/100th meter).

**Order the metric measurements from least to greatest.**

1. 3.3 m; 9,790 mm; 0.612 cm ______________________________

2. 9,400 m; 418 mm; 26,700 cm ______________________________

3. 0.215 m; 265,000 mm; 3,600 cm ______________________________

There are a number of approximate conversions between metric and US length units. These include:

- A meter is about the same length as a yard
- A meter is about three feet long
- A decimeter is about four inches long
- An inch is about 25 millimeters
- A foot contains about 30 centimeters
- A foot contains about 3 decimeters

**Graphing:**

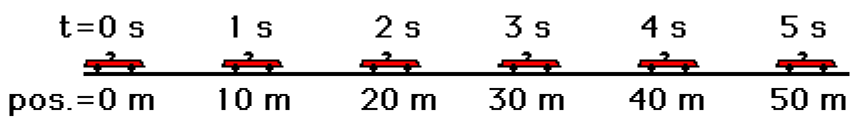
Position vs. Time graphs:

y= mx+b linear equation

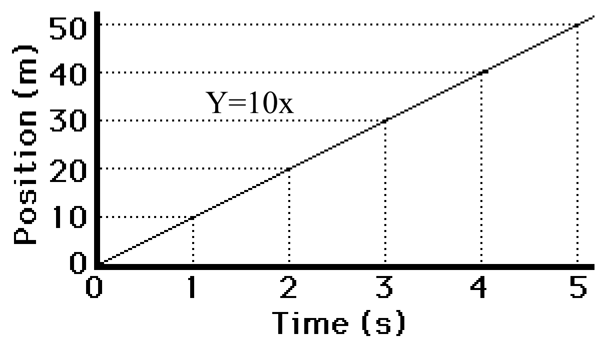
Slope (m): Rate of change on the y-axis with respect to the change on the x-axis. Another way to think of it is Rise/Run. The slope is the same as the velocity of the object that is in motion.

Y-intercept (b): Where the graph intersects the y- axis. For position graphs this is where the object is at time zero (x=0). In our practices the y-intercept will be zero because we will be measuring the distance travelled by our rockets from the origin of launch.

To begin, consider a car moving with a constant, rightward (+) velocity - say of +10 m/s.




If the position-time data for such a car were graphed, then the resulting graph would look like the graph at the right. Note that a motion described as a constant, positive velocity results in a line of constant and positive slope when plotted as a position-time graph.



**Fractions**

How to divide fractions:

*If your friend has half a pie, how many quarter-pies are in that half?* Or, to put this into mathematical notation:

1/2 ÷ 1/4 = ?
(amount we have) ÷ (segment) = (number of portions that are in what we have)

To get the answer, **flip the divisor** (the second fraction) **over, and then multiply the fractions.** (Or, to put it another way, multiply the dividend [the first fraction] by the reciprocal of the divisor [the second fraction].)

In this case, that makes the problem:

$^{1}/2 \times {}^{4}/1 = ?$

We begin by multiplying the numerators:

1 x 4 = 4

And then we multiply the denominators:

2 x 1 = 2

The answer has a numerator of 4 and a denominator of 2. In other words:

1 x 4/2 x 1 =4/2

This fraction can be reduced to lowest terms:

4 ÷ 2/2 ÷ 2 = 2/1 = 2

*There are 2 quarter-pies in a half-pie.*

**Divide. Write your answer as a mixed number in simplest form.**

1. 1/2 ÷ 1/10 =
2. 3/8 ÷ 1/4 =
3. 2/9 ÷ 1/2 =
4. 3/5 ÷ 4/5 =

**Decimals**

How to divide a four digit decimal number by a two digit decimal number
(e.g. 0.424 ÷ 0.8).

- Place the divisor before the division bracket and place the dividend (0.424) under it.

$0.8\overline{)0.424}$

- Multiply both the divisor and dividend by 10 so that the divisor is not a decimal but a whole number. In other words move the decimal point one place to the right in both the divisor and dividend

$8\overline{)4.24}$

- Proceed with the division as you normally would except put the decimal point in the answer or quotient exactly above where it occurs in the dividend. For example:

```
  0.53
8)4.24
  4 0
    24
     0
```

**Divide.**
2 ⁄ 2.4

4 ⁄ 4.4

**Divide. Round to the nearest hundredth.**
8.4 ⁄ 7.0

4.8 ⁄ 2.7

## Cost

The next problem to consider is **cost**. Engineers have to work on a budget, so it is important not to spend more money than we need to. If each part of a rocket costs a certain amount of money, then it is important to try and use as few parts as possible.

Example:

Reggie is asked to build a rocket out of bottles and paper plates. Each bottle costs $1, and each paper plate costs $2. If Reggie's rocket used 10 bottles and 6 paper plates, how much did his rocket cost?

Answer: Cost = 10 bottles + 6 paper plates = 10 x $1 + 6 x $2 = $10 + $12 = **$22**

Reggie's rocket cost $22 to build!

**Price/Unit**

A rate is a form of ratio in which the two terms are in different units. For example price of wheat is $2 for 3 Kilograms (kg), then the rate would be $2 for 3 Kg and the unit of rate would be $/Kg. Similarly if a car goes 100 miles in 1.5 hour, then the rate is 100 miles per 1.5 hour and unit is miles/hr. Note that ratios are usually unit less.

Unit rate is a rate in which the rate is expressed as a quantity of 1. Simply put is rate which has denominator of 1. For example, if a car goes 60 miles in 1 hour, then the unit rate is 60 miles per hour. Other examples are $5 per Kg, 5 meters per second and $80 per barrel.

Unit price is the rate when it is expressed in unit currency like dollar or cent. An example is price of corn is $2 per ounce and price of gas is $5 per gallon. Remember that the price is always the numerator and the unit is the denominator.

If you want to compare the costs of goods, you need to determine what units, from one good to the next, you want to compare. You do this by finding the unit price of each good.

Examples:

1. Scott really likes chocolate bars, and wants to get the best deal possible on them. Is it better for him to buy 3 for $2.25 or each one at $ .79 each.

a) Determine the cost of each chocolate bar by dividing $ 2.25 by 3
$2.25=$0.75
It is cheaper for Scott to buy 3 chocolate bars for $2.25 because they work out to be $ .75 each compared to $ .79 which they would cost if he bought them separately.

2. Becky eats cereal for breakfast every morning. Is it better for her to buy a 550 g box of cereal for $2.50, or a 1 kg box for $5.00?

a) Work out the cost per gram of the 550 g box (divide by 550)

550 g costs $ 2.50

1 g costs
$\frac{\$2.50}{550} = \$0.004545$

1 g of cereal from the 550 g box costs $ 0.004545

b) Work out the cost per gram of the 1 kg (1000 g) box

1000 g costs $ 5.00

1 g costs
$\frac{\$5.00}{1000} = \$0.005$

1 g of cereal from the 1000 g box costs $ 0.005

Working out the cost per gram of each box of cereal, Becky realizes that it is a better deal for her to buy the 550 g box.

Best Deal **Challenge:**

**Problem 1...**
1 gallon of milk for $2.25 ($2.25 per gallon - not the best deal)
2 gallons of milk for $4.00 ($2.00 per gallon - the best deal)
3 gallons of milk for $6.50 ($2.17 per gallon - not the best deal)

**Problem 2...**
1 pack of gum for $0.50 ($0.50 per pack - the best deal)
2 packs of gum for $1.25 ($0.63 per pack - not the best deal)
5 packs of gum for $5.00 ($1.00 per pack - not the best deal)

**Problem 3...**
1 video game for $30 ($30.00 per game - not the best deal)
3 video games for $70 ($23.33 per game - not the best deal)
5 video games for $115 ($23.00 per game - the best deal)