Engineering GatorTRAX

Projectile Motion Module
Advanced Level

Designed in accordance with Tau Beta Pi MindSET standards
By ***University of Florida Engineering Ambassadors, 2009***

Trigonometry

Definition: Trigonometry is the relationship between lines and angles and often involves triangles. Trig functions include sines, cosines, and tangents.

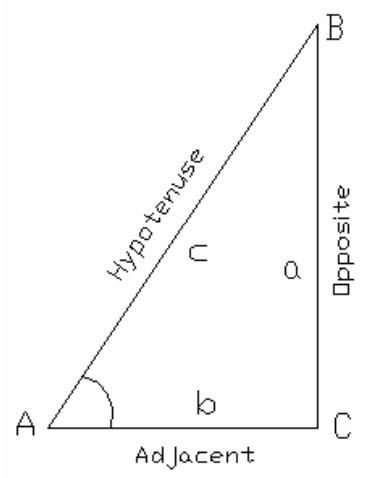
Sin(A)= Opposite/Hypotenuse

Cos(A)= Adjacent/Hypotenuse

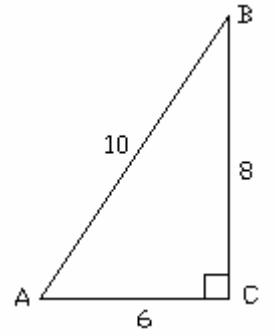
Tan(A)= Opposite/Adjacent

SOH CAH TOA (pronounced soak-uh-toe-uh) is a good way to remember these equations. It is important to specify which angle you are using in a trig function.

*Note: These three trig functions only work for right triangles which are triangles that have one angle equal to 90°.



Examples: Use the figure below to answer the questions.



1) Find sin(A), cos(A), and tan(A)

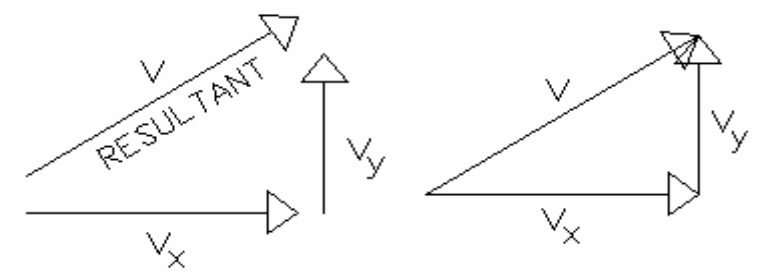
2) Find sin(B), cos(B), and tan(B)

3) Can you find sin(C), cos(C), and tan(C)? Why or why not?

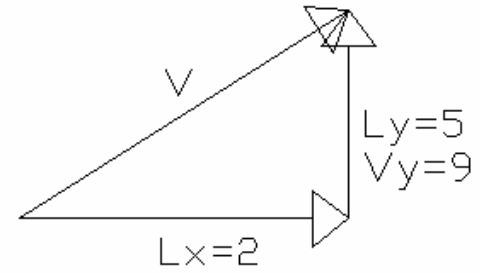
Vectors

Definition: a line that has a magnitude (length) and a specified direction. Vectors can be broken up into vertical (y) and horizontal (x) components. Add the x and y components to find the resultant. The resultant will be the hypotenuse of a 90° triangle. Vector magnitudes are proportional to their lengths.

$$V/L = V_x / L_x = V_y / L_y$$

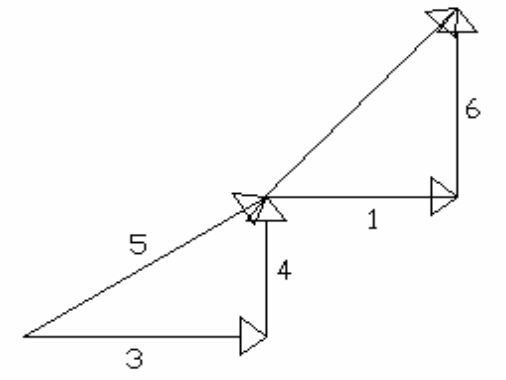


Example: Find the magnitude of the resultant in the figure below. (Leave the square root as it is).



Adding Vectors: when you add two vectors, add all the x components and then all the y components. This will give you the x and y components for the resultant of all the added vectors.

Example: Add the vectors below. They have been broken up into components for you.



Measurement

Units for measurement: Important measurements for us today are going to be weight , length, and time. There are 2 standardized systems of measurement. The S.I (International Standard) units, also known as metric units, are used worldwide especially in the world of science and The other system is known as Imperial System and is specific to the United States and not often used worldwide. These units include feet

Weight:

1 lb	=	.0005 ton	=	.454 kg	=	454 g
2000 lb	=	1 ton	=	908 kg	=	908000 g
.225 lb	=	.000113 ton	=	102 kg	=	102 g
2.20 lb	=	.0011 ton	=	1 kg	=	1000 g
.00220 lb	=	.0000011 ton	=	.001 kg	=	1 g

Length: You probably measure length in feet (ft), but other units that are used in engineering include inches (in), centimeters (cm), and meters (m). A meter is an S.I unit while the rest are all Imperial units.

1 ft	=	12 in.	=	30.48 cm	=	.3048 m
.083 ft	=	1 in.	=	2.54 cm	=	.0254 m
.0328 ft	=	.394 in.	=	1 cm	=	.01 m
3.28 ft	=	39.4 in.	=	100 cm	=	1 m

if a chimpanzee weighs 175 pounds how many grams does he weigh? (use a calculator)

_____

If a rocket travels 5 feet, how many inches does it travel? How many meters?

_____

The metric system has prefix modifiers that are multiples of 10.

Prefix	Symbol	Factor Number	Factor Word
Kilo	k	1,000	Thousand
Hecto	h	100	Hundred
Deca	da	10	Ten
Deci	d	0.1	Tenth
Centi	c	0.01	Hundredth
Milli	m	0.001	Thousandth

Inequalities, Equations, and Ratios

An equation is a mathematical statement that has two expressions separated by an equal sign (=). The expression on the left side of the equal sign has the same value as the expression on the right side.

One or both of the expressions may contain variables. Solving an equation means manipulating the expressions and finding the value of the variables.

Example: Solve the equation: $8x - 2 = 14$

To keep both sides of an equation equal, we must do exactly the same thing to each side of the equation.

First, add 2 to each side of the equation:	8x - 2 **+ 2**	=	14 **+ 2**
	8x	=	16
Now to solve, we divide both sides by 8:	8x / 8	=	16 / 8
	x	=	2

Now we know that x=2! You can check your work by substituting the value you got for x back into the original equation:

8x - 2	=	14
8*2 - 2	=	14
16 - 2	=	14
14	=	14

Since the equation simplified to 14 = 14, you know you got the right answer for x!

An inequality is similar to an equation. There are two expressions separated by a symbol that indicates how one expression is related to the other.

In an equation such as 7x = 49, the = sign indicates that the left side and the right side are equal.

In an inequality, such as 7x > 49, the > sign indicates that the left side is greater than the right side.

To solve the inequality, we follow the same rules that we did for equations.

	7x	>	49
Divide both sides by 7:	7x / 7	>	49 / 7
Now we know the value for x:	x	>	7

The inequality x > 7 means that x can be any number that and it is larger than 7, and x cannot be equal to or less than 7.

The "less than" symbol (<) may also be seen in inequalities.

Ratios represent how one quantity is related to another quantity.
A ratio may be written as A:B or A/B or by the phrase "A to B".
A ratio of 1:5 says that the second quantity is five times as large as the first.

The following steps will allow a ratio to be determination if two numbers are known.
Example: Determine the ratio of 24 to 40.

- Divide both terms of the ratio by the greatest common factor (24/8 = 3, 40/8=5)
- State the ratio. (The ratio of 24 to 40 is 3:5)

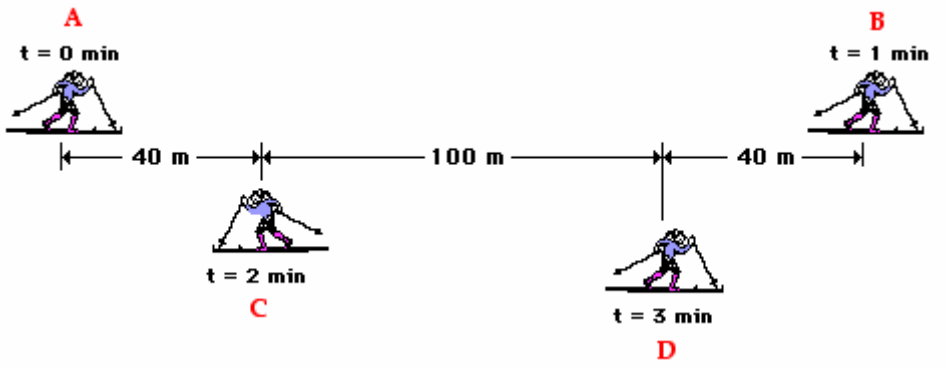
Velocity: Velocity is a vector which refers to "the rate at which an object changes its position." Imagine a person moving rapidly - one step forward and one step back - always returning to the original starting position. While this might result in a frenzy of activity, it would result in a zero velocity. Because the person always returns to the original position, the motion would never result in a change in position. Since velocity is defined as the rate at which the position changes, this motion results in zero velocity. If a person in motion wishes to maximize their velocity, then that person must make every effort to maximize the amount that they are displaced from their original position. Every step must go into moving that person further from where he or she started.

The Gator Engineer walks 4 meters East, 2 meters South, 4 meters West, and finally 2 meters North. The entire motion lasted for 24 seconds. Determine the velocity and speed.

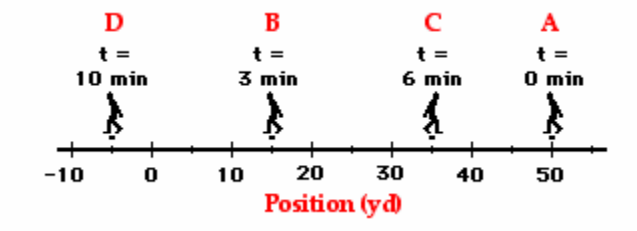
The Gator Engineer walked a distance of 12 meters in 24 seconds. This means her average speed was: 12 meters / 24 seconds = 0.50 m/s.

However, since her displacement is 0 meters, her average velocity is 0 m/s. Remember that the displacement refers to the change in position and the velocity is based upon this position change. In this case of the teacher's motion, there is a position change of 0 meters and thus an average velocity of 0 m/s.

Challenge:
Use the diagram to determine the average speed and the average velocity of the skier during these three minutes.



What is the coach's average speed and average velocity?



Graphing:

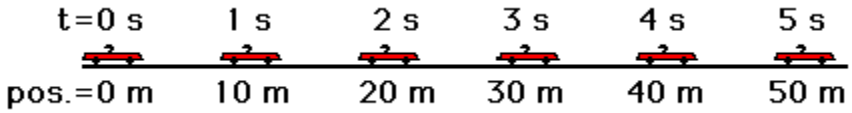
Position vs. Time graphs:

y= mx+b linear equation

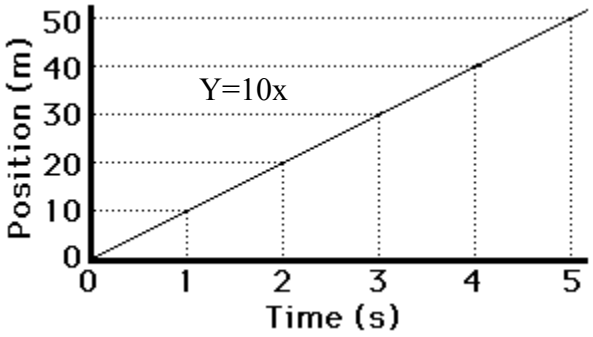
Slope (m): Rate of change on the y-axis with respect to the change on the x-axis. Another way to think of it is Rise/Run. The slope is the same as the velocity of the object that is in motion.

Y-intercept (b): Where the graph intersects the y- axis. For position graphs this is where the object is at time zero (x=0). In our practices the y-intercept will be zero because we will be measuring the distance travelled by our rockets from the origin of launch.

To begin, consider a car moving with a constant, rightward (+) velocity - say of +10 m/s.



If the position-time data for such a car were graphed, then the resulting graph would look like the graph at the right. Note that a motion described as a constant, positive velocity results in a line of constant and positive slope when plotted as a position-time graph.



Cost

The next problem to consider is **cost**. Engineers have to work on a budget, so it is important not to spend more money than we need to. If each part of a rocket costs a certain amount of money, then it is important to try and use as few parts as possible.

Example:

Reggie is asked to build a rocket out of bottles and paper plates. Each bottle costs $1, and each paper plate costs $2. If Reggie's rocket used 10 bottles and 6 paper plates, how much did his rocket cost?

Answer: Cost = 10 bottles + 6 paper plates = 10 x $1 + 6 x $2 = $10 + $12 = **$22**

Reggie's rocket cost $22 to build!

Price/Unit

A rate is a form of ratio in which the two terms are in different units. For example price of wheat is $2 for 3 Kilograms (kg), then the rate would be $2 for 3 Kg and the unit of rate would be $/Kg. Similarly if a car goes 100 miles in 1.5 hour, then the rate is 100 miles per 1.5 hour and unit is miles/hr. Note that ratios are usually unit less.

Unit rate is a rate in which the rate is expressed as a quantity of 1. Simply put is rate which has denominator of 1. For example, if a car goes 60 miles in 1 hour, then the unit rate is 60 miles per hour. Other examples are $5 per Kg, 5 meters per second and $80 per barrel.

Unit price is the rate when it is expressed in unit currency like dollar or cent. An example is price of corn is $2 per ounce and price of gas is $5 per gallon. Remember that the price is always the numerator and the unit is the denominator.

If you want to compare the costs of goods, you need to determine what units, from one good to the next, you want to compare. You do this by finding the unit price of each good.

Examples:

1. Scott really likes chocolate bars, and wants to get the best deal possible on them. Is it better for him to buy 3 for $2.25 or each one at $ .79 each.

a) Determine the cost of each chocolate bar by dividing $ 2.25 by 3
$2.25=$0.75
It is cheaper for Scott to buy 3 chocolate bars for $2.25 because they work out to be $ .75 each compared to $ .79 which they would cost if he bought them separately.

2. Becky eats cereal for breakfast every morning. Is it better for her to buy a 550 g box of cereal for $2.50, or a 1 kg box for $5.00?

a) Work out the cost per gram of the 550 g box (divide by 550)

550 g costs $ 2.50

1 g costs
$\frac{\$2.50}{550} = \$0.004545$

1 g of cereal from the 550 g box costs $ 0.004545

b) Work out the cost per gram of the 1 kg (1000 g) box

1000 g costs $ 5.00

1 g costs
$\frac{\$5.00}{1000} = \$0.005$

1 g of cereal from the 1000 g box costs $ 0.005

Working out the cost per gram of each box of cereal, Becky realizes that it is a better deal for her to buy the 550 g box.

Best Deal **Challenge:**

Problem 1...
1 gallon of milk for $2.25 ($2.25 per gallon - not the best deal)
2 gallons of milk for $4.00 ($2.00 per gallon - the best deal)
3 gallons of milk for $6.50 ($2.17 per gallon - not the best deal)

Problem 2...
1 pack of gum for $0.50 ($__.__ per game)
2 packs of gum for $1.25 ($__.__ per game)
5 packs of gum for $5.00 ($__.__ per game)

Problem 3...
1 video game for $30 ($__.__ per game)
3 video games for $70 ($__.__ per game)
5 video games for $115 ($__.__ per game)

CHALLENGE:

Percy and Maria are competing against each other in a rocket-building competition. They are each allowed to use as many bottles, adhesives, and wings/fins as they wish, but each comes at a price:

Adhesive	$1
wings/fins	$3
Bottles	$5

The data from each of their rockets is below. Whose bridge was more cost-effective?

	Percy's rocket	Maria's rocket
# of adhesives	18	17
# of wings/fins	8	13
# of bottles	3	4
Cost ($)		