

Engineering GatorTRAX

Structural Engineering Module Advanced Level

*Designed in accordance with Tau Beta Pi MindSET standards
By American Society of Civil Engineers, University of Florida Chapter, 2009*



Trigonometry

Definition: Trigonometry is the relationship between lines and angles and often involves triangles. Trig functions include sines, cosines, and tangents.

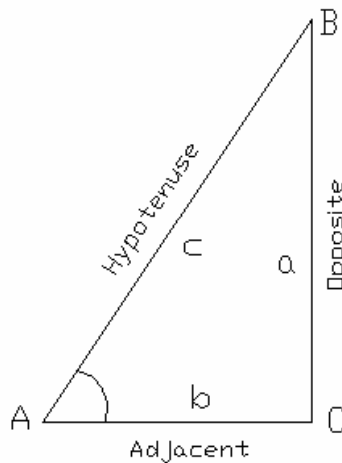
$$\sin(A) = \text{Opposite}/\text{Hypotenuse}$$

$$\cos(A) = \text{Adjacent}/\text{Hypotenuse}$$

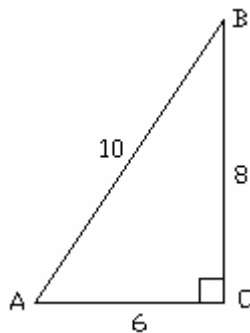
$$\tan(A) = \text{Opposite}/\text{Adjacent}$$

SOH CAH TOA (pronounced soak-uh-toe-uh) is a good way to remember these equations. It is important to specify which angle you are using in a trig function.

*Note: These three trig functions only work for right triangles which are triangles that have one angle equal to 90° .



Examples: Use the figure below to answer the questions.

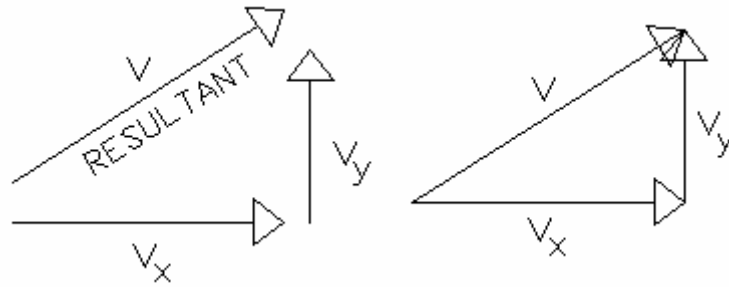


- 1) Find $\sin(A)$, $\cos(A)$, and $\tan(A)$
- 2) Find $\sin(B)$, $\cos(B)$, and $\tan(B)$
- 3) Can you find $\sin(C)$, $\cos(C)$, and $\tan(C)$? Why or why not?

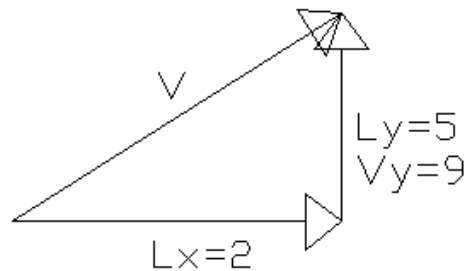
Vectors

Definition: a line that has a magnitude (length) and a specified direction. Vectors can be broken up into vertical (y) and horizontal (x) components. Add the x and y components to find the resultant. The resultant will be the hypotenuse of a 90° triangle. Vector magnitudes are proportional to their lengths.

$$V/L = V_x/L_x = V_y/L_y$$

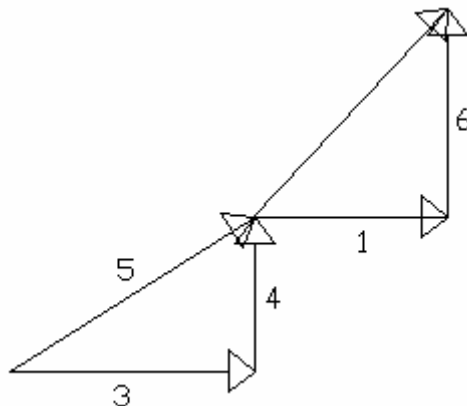


Example: Find the magnitude of the resultant in the figure below. (Leave the square root as it is).



Adding Vectors: when you add two vector, add all the x components and then all the y components. This will give you the x and y components for the resultant of all the added vectors.

Example: Add the vectors below. They have been broken up into components for you already.



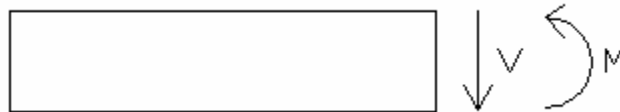
Forces

Definition: a force is that which can cause an object with mass to accelerate. Forces have both magnitude and direction which means they are vectors.

$$F = \text{mass} * \text{acceleration}$$

Shear

Definition: An internal vertical force that acts parallel to the face of the cross section. Shear is denoted by V in the figure below.



Moments

Definition: a force that causes rotation around a pivot point. A moment is positive if it causes a counterclockwise rotation and negative if it causes clockwise rotation. The symbol for moment is shown in the figure above.

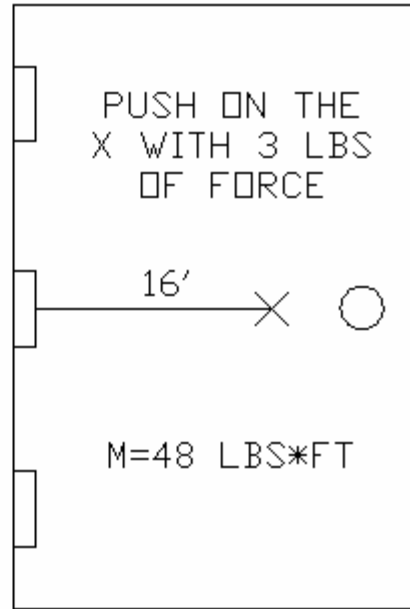
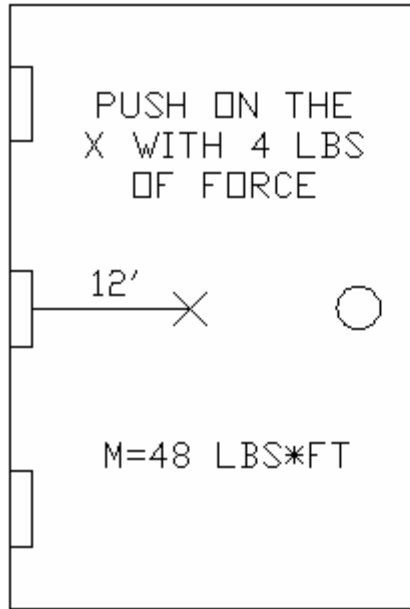
Moment Arm: the distance from the pivot point to the place where the force is exerted on the object.

$$M = r * F$$

M is the moment (lbs*ft or N*m), r is the moment arm (ft or meters), F is the force applied (lbs or Newtons)

*Note: r and F are perpendicular to each other.

Push on a door near the hinge and then try to push on it near the door knob. Notice that it is harder to move the door when you push near the hinge. This is because the moment arm is shorter near the hinge so you have to push harder to produce the same moment you get from pushing near the door knob.



The door requires the same moment to be moved no matter what, but by pushing further from the hinge you can exert a smaller force and it is easier for you to move it.

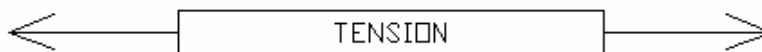
Axial Forces

Definition: axial forces are applied perpendicular to the face of the cross section and cause normal stresses.

Compressive Stress: when a force is applied to some material that causes it to shorten or become compact the object is under compression. Concrete is good at handling compressive forces. Compression is often represented as a negative value.



Tensile Stress: when a force is applied to some material that causes it to stretch or become longer and more slender, the object is in tension. Steel is good at handling tensile forces. Rope and string can only be in tension (never in compression). Tension is often represented as a positive value.



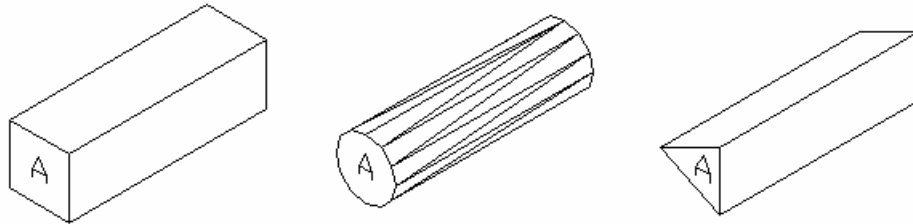
Stress = force/area

$$\sigma = F/A$$

units: lbs/in² or N/m²

The area in this equation is the area of the cross section of a piece of material. The cross section is what you see when you cut the material.

For example, the cross sectional area of the middle figure below is πr^2 where r is the radius of the circle and π is an irrational constant that we often round to 3.14.



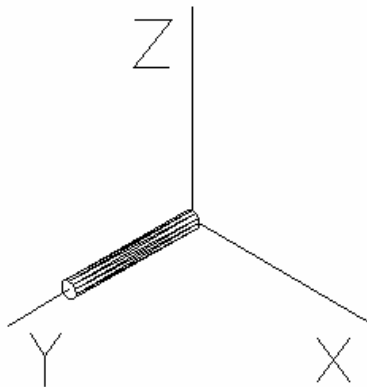
Inertia

Moment of Inertia: an object's *resistance* to changes in its rotation rate. The moment of inertia changes when you rotate the object around different axes.

$$I=MR^2$$

M = mass, R = distance from the center of mass of the object to the axis it is rotating around, I = moment of inertia

Center of Mass: the point in an object where it can be balanced. For example, a seesaw has a center of mass right in the middle. You can balance the seesaw because it has a pivot point right at the center of mass.



When talking about the moment of inertia, you have to specify which axis you are talking about. It doesn't make sense to say "What is the moment of inertia of the cylinder?" It does make sense to say "What is the moment of inertia of the cylinder about the Y axis?"

The cylinder in the figure above has a smaller moment of inertia when it is rotated around the Y axis than when it is rotated around the X or Z axis because R is smaller when it is measured from the center of the cylinder to the Y axis.

Polar Moment of Inertia: an object's ability to resist torsion which is twisting. This is often represented by the symbol J .

Look at the piece of wood. When you rest it on a table and try to break it over the edge, it is much easier to break when the longest side of the cross section is pressed flat against the table. Try breaking it with the shortest side now and note which way is easier. This is because of the moment of inertia.

Example Questions:

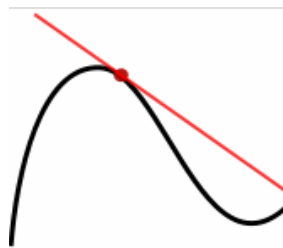
The cylinder in the figure above has a radius of 3 m, a length of 20 m, and a mass of 200kg.

- 1) What is the moment of inertia about the Y axis?
- 2) What is the moment of inertia about the X axis?
- 3) What is the moment of inertia about the Z axis?

Derivatives

Definition: the rate of change of an output function with respect to an input function. In other words, it is the rate of change of y with different values of x .

Line of Tangency: a line that is drawn to touch one point on a curve. The angle of the line matches the angle of the curve at that point.



If you have a graph and draw tangency points at several locations, then you will be able to graph the derivative. Each tangent line has a slope so, to draw the derivative, take a note of what the slope is at each point then graph it.

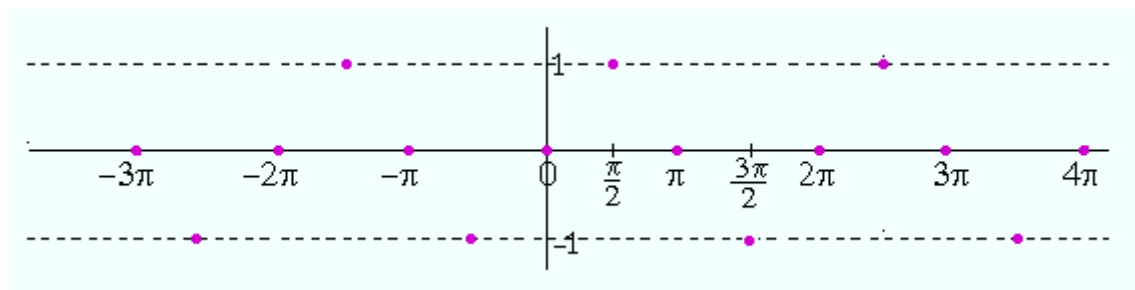


Figure A sine graph

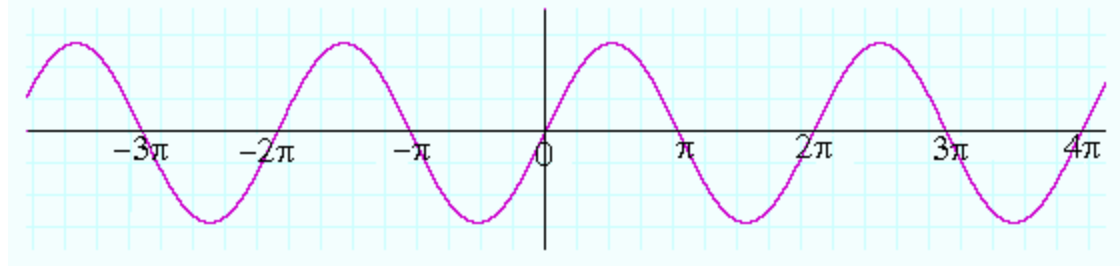
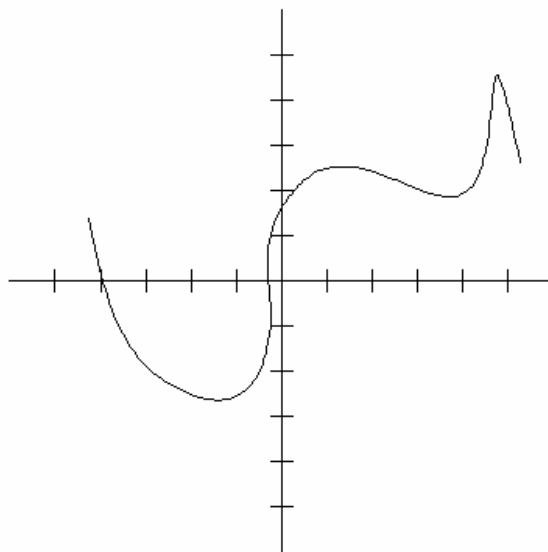


Figure B sine graph with points connected

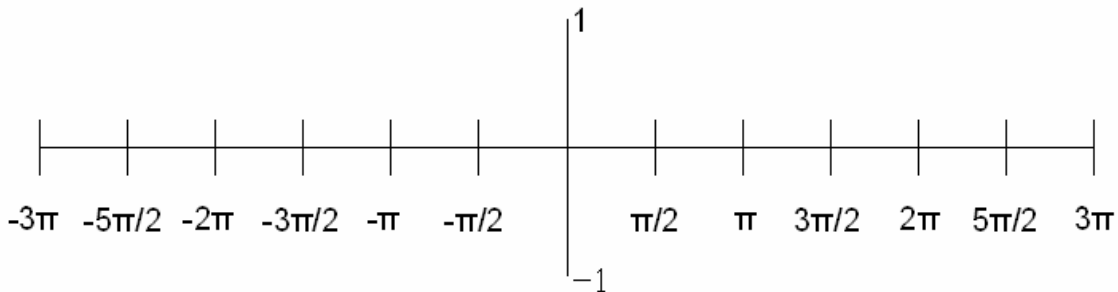
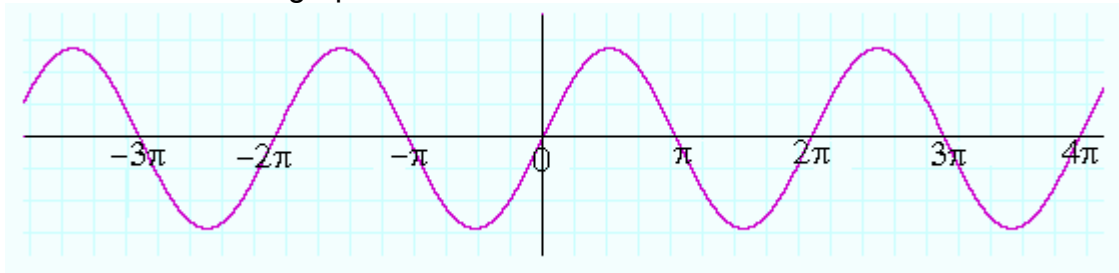
This is a sin graph. The maximum y value is 1 and it is where the graph changes from increasing to decreasing and the minimum y value is -1 and is where the graph changes from decreasing to increasing (See Figure A). The slope at the max and min values is 0. This means the derivative of this graph would have y values of 0 at odd intervals of $\pi/2$ (ex. $\pi/2, 3\pi/2, 5\pi/2$). Notice that everywhere the graph is increasing (not where it is positive, but where it is increasing), the slopes are positive and everywhere the graph is decreasing the slopes are negative.

Concavity: every point on the graph can be labeled as concave up, concave down, or a point of inflection. Concave up means the graph is shaped like a bowl that is right side up and concave down means it is shaped like an upside down bowl. The sin graph is concave up from π to 2π and is concave down from 2π to 3π (See Figure B). A point of inflection is where the graph changes from concave up to concave down or visa versa. All integer multiples of pi are points of inflection (ex. $\pi, 2\pi, 3\pi$).

Examples: Each dash below represents 1 unit. The x axis is horizontal and the y axis is vertical. How many max points are on this graph? How many min points? Draw tangent lines at x values of -3, -1.5, -1, 0, 2, 3, 4, and 5.



Draw the derivative graph for sin theta.



Power Rule: when you have an equation with exponents in it you can easily take the derivative using the equation below. n is the original exponent.

$$n \cdot x^{n-1}$$

The derivative of sin theta = cos theta and the derivative of cos theta = -sin theta

Examples: Find the derivatives of the functions below

$$y = x^2$$

$$y = 6x^5$$

$$y = 4x^3 + 8x^6$$

$$y = \cos(45)$$

$$y = \sin(82)$$

Integrals

Definition: The area under a bounded portion of a curve. If there are exponents in the equation you can find its integral by using the equation below. n is the original exponent and C is some constant. Trig rules are also below.

$$\left(\frac{1}{(n+1)}\right) \cdot x^{(n+1)} + C \quad \text{int}(\cos \theta) = \sin \theta \quad \text{int}(\sin \theta) = -\cos \theta$$

Examples: Find the integral of the sin graph in the example above from $x=0$ to $x=\pi$. Find the integrals for the equations below

$$y = x^3$$

$$y = 9x^2$$

$$y = 9x^7 + 5x^4$$

Shear, moments, and stresses are very important to a structure's stability. You will now do a project that will reinforce everything you have learned today.